The use of CLP(FD) extended with CHRs for Qualitative Spatial Reasoning and Qualitative Robot Navigation

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Abstract

With the aim of simulating the human spatial reasoning capabilities, several qualitative models have been developed in the recent years. Most of these models deal with one or, at most, two aspects of space and, the more cognitive models, simplify spatial objects to points. In this paper we present a new approach that uses Constraint Logic Programming instantiated to Finite Domains extended with Constraint Handling Rules as tool for integrating several spatial aspects into the same qualitative model using point as well as extended objects as primitive of reasoning. The qualitative concepts of orientation, position (i.e. orientation + distance), and cardinal directions on points and extended objects are successfully integrated using this approach. As example, the integration of the concept of orientation into the model is explained in this paper. The resulting model is applied to build a demonstrator of a Qualitative Navigation Simulator on the structured environment of the city of Castellón.

Keywords: Spatial Reasoning, Qualitative Reasoning, Constraint-Based Reasoning, Qualitative Robot Navigation.

1 Introduction

An intelligent robot should carry out spatial reasoning for solving problems dealing with entities occupying space. If we attempt to simulate the human spatial reasoning process, it is necessary to deal with descriptions of positions of objects which contain information such as "the cinema is to your *right*, *far* from here and to the *north-west* of the city".

In the last years, many qualitative spatial models have been developed to manage properly the imprecise knowledge about different aspects of space. For the concept of orientation, there exist mainly three models based on orthogonal projections [15] and [20] and non orthogonal projections [25] (figure 1a) and three models not based on projections [12, 13], [11] and [18] (figure 1b). In those models based on projections, the relative orientation among objects is obtained by drawing orthogonal or no orthogonal projections, and then reasoning in one-dimension by using the Allen's temporal reasoning [1]. In those models not based on projections, the 2-dimensional space is divided into qualitative regions by means of Reference Systems (RSs) which are centered on the referenced object. The set of models not based on projections are considered more cognitive due mainly to two reasons: (1) people do not reason

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about spatial orientation by doing projections on "inexistent" external axes. They rather think about egocentric RSs based on the asymmetry of the human body (front/back) —strongly influenced by the vision system— and both arms (right/left) [18]; and (2) in models based on orthogonal projections, it is possible to infer inconsistent knowledge (for instance, two overlapped relationships between the projections of objects in both axes, X and Y, do not imply that the two objects overlap in the two-dimensional space). Due to all these reasons we are more interested in models not based on projections.

However, on the other hand, although the qualitative models based on projections consider extended objects as primitive of reasoning —sometimes objects are approximated to rectangles— qualitative models not based on projections always simplify objects to points.

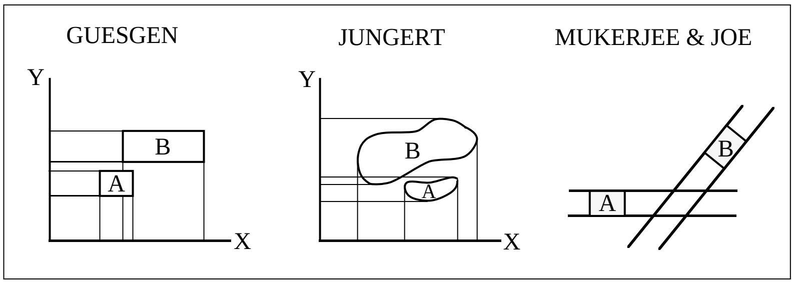
The qualitative orientation models not based on projections have also been extended with the concept of distance in the following models [27], [19] and [2] (see [6] for further details).

As we have pointed out in this brief state of the art, there exist several qualitative models to represent and reason about orientation and positional information. However, the integration of orientation, cardinal directions and distance into the same qualitative model is a problem which remains unsolved. That is, there is no general model to reason with all these spatial aspects in a uniform way.

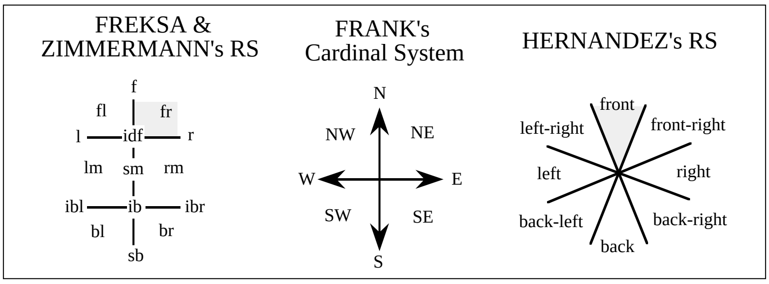
Therefore the objectives of our work are the following:

(1) the integration of several aspects of space into the same qualitative model,
(2) the use of point and extended objects as primitive of reasoning, and
(3) the development of a demonstrator of the application of the spatial reasoning model to Qualitative Navigation.

The bases for the integration of several spatial aspects in the Spatial Reasoning (SR) domain have been inspired in the Temporal Reasoning (TR) field where point and interval algebra and metric information have been successfully combined using constraint satisfaction techniques [23].



a)



b)

Figure 1. Qualitative orientation models a) based on projections and b) not based on projections.

2 Bases for the integration of several temporal aspects in the Temporal Reasoning field

Many forms of temporal relationships have been integrated thanks to consider them instances of the Constraint Satisfaction Problem (CSP). Generate and test, and backtracking are algorithms which solve the CSP, although in a very inefficient way (these algorithms have an exponential cost). Research in the field tries to improve efficiency of the backtracking algorithm (a review of the state of the art can be found in [24] and [21]). A set of these algorithms modify the search space before the search process starts, to make the search process easier. They are called algorithms which improve consistency. These algorithms are based on the idea of making explicit the implicit constraints by means of the constraint propagation process. Unfortunately the complete constraint propagation process is also hard, therefore the process is approximated by local constraint propagation, as path consistency. If the constraint graph is complete (that is, there is a pair of arcs, one in each direction, between every pair of nodes) it suffices to repeatedly compute paths of length two at most. This means that for each group of three nodes (i,k,j) we repeatedly compute the following operation until a fix point is reached [14]:

$$c_{ij} := c_{ij} \oplus c_{ik} \otimes c_{kj} \tag{1}$$

This operation computes the composition ($\otimes$) of constraints between nodes *ik* and *kj* and the intersection ($\oplus$) of the result with constraints between nodes *ij*. The complexity of this algorithm is $O(n^3)$, where *n* is the number of nodes in the constraint graph (that is, the number of objects involved in the reasoning process) [22].

Constraint Handling Rules (CHRs) are a tool which helps to write the above algorithm. They are an extension of the Constraint Logic Programming (CLP) which facilitate the definition of constraint theories and algorithms which solve them. They facilitate the prototyping, extensions, specialization and combination of Constraint Solvers (CSs) [14]. The part $\left(c_{ik} \otimes c_{kj}\right)$ of the formula will be implemented by propagation CHRs which are of the form:

$$H_1,\ldots,H_i \text{ ==> } G_1,\ldots,G_j \mid B_1,\ldots,B_k \qquad \left(i > 0, j \geq 0, k \geq 0\right)$$

Which means that if a set of constraints matches the head ($H_1,...,H_i$) of a propagation CHR and the guards ($G_1,\ldots,G_j$) are satisfied, then the set of constraints $B_1,\ldots,B_k$ is added to the set of initial constraints($H_1,...,H_i$). These constraints are redundant but might cause further simplifications.

The part of the formula which refers to $\left(c_{ij} \oplus \ldots\right)$ will be implemented by simplification CHRs which are of the form:

$$H_1,\ldots,H_i \text{ <=> } G_1,\ldots,G_j \mid B_1,\ldots,B_k \qquad \left(i > 0, j \geq 0, k \geq 0\right)$$

Which means that if a set of constraints ($H_1,...,H_i$) matches the head of a simplification CHR and the guards ($G_1,\ldots,G_j$) are satisfied, the set of constraints $H_1,...,H_i$ is substituted by the set of constraints $B_1,\ldots,B_k$. The set of constraints $B_1,\ldots,B_k$ is simpler than the set of constraints $H_1,...,H_i$ and preserves logical equivalence.

CHRs are included as a library in ECLiPSe [3].

3 Bases for the integration of several spatial aspects in the Spatial Reasoning field

We have divided the reasoning process into two parts: the Basic Step of the Inference Process (BSIP) and the Full Inference Process (FIP). For those models not based on projections, the BSIP can be defined in general terms such as: given a spatial relationship between object A with respect to (wrt) a RS, RS1, and another spatial relationship between object B wrt another RS, RS2, being object A part of the RS2, the BSIP consists of obtaining the spatial relationship of object B wrt the RS1. The RS will be different depending on the model. When more relationships among several spatial landmarks are provided, then the FIP is necessary. It consists of repeating the BSIP as many times as possible, with the initial information and the information provided by some BSIP, until no more information can be inferred.

To accomplish the integration of orientation, position (orientation + distance), and cardinal direction information into the same model we have used the following three steps which define:

(1) the representation of the spatial aspect to be integrated;
(2) the BSIP for each represented spatial aspect; and
(3) the FIP for this spatial aspect.

As an example, in the next sections, these three steps are explained for the concept of orientation information. For a detailed explanation of the integration of the other concepts into the same model, we refer to [4,5,6,7,8,9,10].

3.1 Qualitative Orientation Information

3.1.1 The representational model

For representing the two-dimensional orientation information, the Freksa and Zimmermann's model [12, 13] has been chosen. The orientation in their RS is described by two points, *a* and *b*, which defines the left/right dichotomy and can be interpreted as the direction of movement. The coarse RS also includes the perpendicular line by the point *b*. The point *b* defines the first front/back dichotomy (1F/B) and can be seen as the straight line which joins our shoulders. This coarse RS divides the space into 9 qualitative regions. A finer distinction could be made in the back regions by drawing the perpendicular line by the point *a*. In this case, the space is divided into 15 qualitative regions (figure 2 a). The point *a* defines the second front/back dichotomy (2F/B) of the RS. An iconical representation of the fine RS and the names of the regions are shown in figure 2 b). The information which can be represented by this RS is the qualitative orientation of a point object, *c*, wrt the RS formed by the point objects *a* and *b*, that is, *c wrt ab*.

The relationship *c wrt ab* can also be expressed in other five different ways: *c wrt ba* , *a wrt bc* , *a wrt cb* , *b wrt ac* and *b wrt ca* which are the result of applying the *inverse* (INV), *homing* (HM), *homing-inverse* (HMI), *shortcut* (SC) and *shortcut-inverse* (SCI) operations, respectively (figure 2 c). In our model these operations have been represented as facts of the PROLOG database. For instance, the inverse of the spatial relationship "left-front" is the spatial relationship "back-right" (*inv(lf,[rb]).*).

An important feature of these operations (which is not explained in [12, 13]) is their idempotent property. An operation is idempotent of level two if it is necessary to apply the operation twice to the original relationship to get the original relationship again. The INV, HMI, and SC operations are idempotent of level two. For instance, INV(INV(c wrt ab))=c wrt ab. An operation is idempotent of level three if it is necessary to apply the operation three times to the

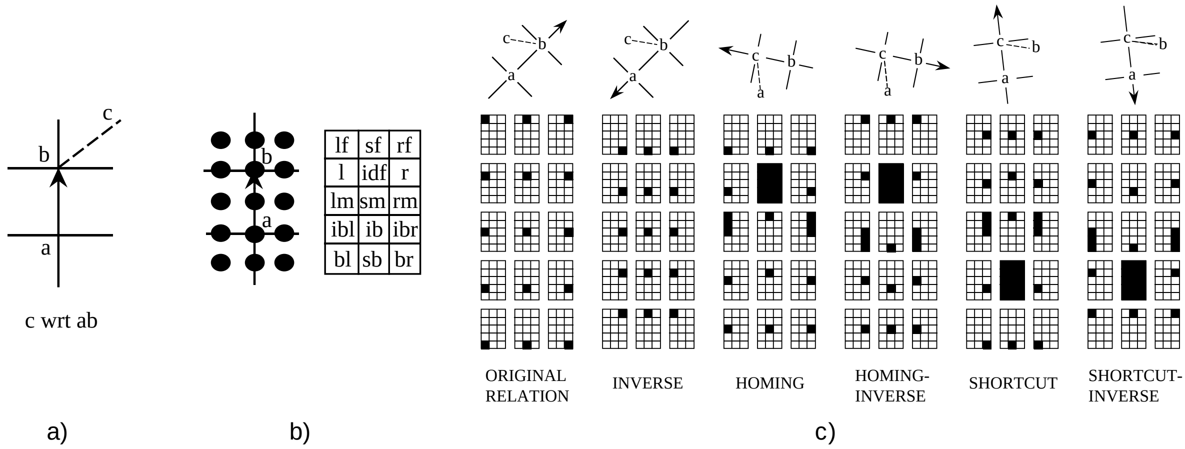


Figure 2. a) The fine RS and b) its iconical representation;
c) Operations to represent by different ways the information included in *"c wrt ab"*.

original relationship to get the original relationship again. The HM and SCI operations are idempotent of level three. For instance, HM(HM(HM(c wrt ab)))=c wrt ab. The operations which are idempotent of level three have another property: the application of the HM operation twice is equivalent to the application of the SCI operation once (i.e. SCI(HM(c wrt ab))= c wrt ab), and the application of the SCI operation twice is equivalent to apply the HM operation once (i.e. HM(SCI(c wrt ab))= c wrt ab). These operations and their properties will be used for the definition of the FIP for orientation information.

3.1.2 The Basic Step of the Inference Process

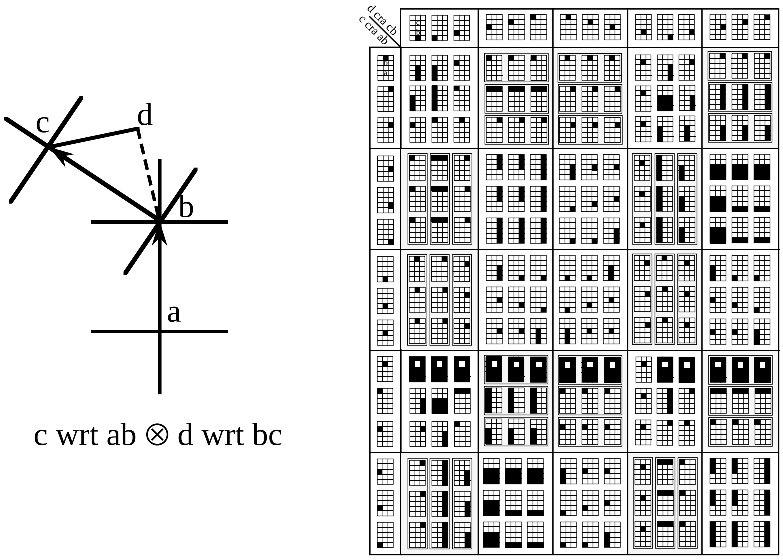
The BSIP for the qualitative spatial orientation is defined such that "given the relationships *c wrt ab* and *d wrt bc*, we want to obtain the relationship *d wrt ab*" (figure 3 a). In [13] two inference tables of 9×9 to define this BSIP are introduced. One table corresponds to the coarser division of the space into 9 regions and the other one to the inference between the relations in which the coarse back relations have been split up. These tables include the composition between the 15 qualitative regions in which the fine RS divides the space, although in an implicit way. In that sense, we say that these inference tables are complete. However, we make explicit the implicit information included in these tables by the definition of the 15×15 inference table shown in figure 3 b. Disjunctions of relationships are represented by more than a black square on the iconical figure. For details of how to obtain it, we refer to [4].

This composition table has been implemented in our approach by facts of the PROLOG database (for example, *composition(sf, sb, [sm, ib, sb])*).

3.1.3 The Full Inference Process

The above explained orientation relationships (*c wrt ab*) are treated in our model as tertiary constraints $\{c_{c,ab}(X_c, X_a, X_b)\}$ which relate triples of variables X_c, X_a, X_b, (1≤a<b<c≤n). They are seen as an instance of the CSP. The operation (1) to compute path consistency is redefined for tertiary constraints so that:

$$c_{d,ab} := c_{d,ab} \oplus c_{c,ab} \otimes c_{d,bc} \tag{2}$$



a) b)

Figure 3. a) The orientation BSIP and b) its composition table.

The orientation constraint $c_{c,ab}$ is represented by the predicate *ctr(C,A,B,Rel)*, where *A*, *B* and *C* are variables which refer to spatial objects and *Rel* is a list which contains a disjunction of orientation relationships between the three spatial objects.

The intersection part of the formula ("⊕") is implemented with the following simplification CHR:

ctr(C,A,B,R1), ctr(C,A,B,R2) <=> intersection(R1,R2,R3) | ctr(C,A,B,R3) (3)

If the constraint graph were complete, no more simplification CHRs would be needed. However, usually there is no relationship between every object in space. This lack is supplied with the definition of more simplification rules by using the five operations above mentioned. For instance:

ctr(B,C,A,R1'), ctr(C,A,B,R2) <=> hm(R1',R11), intersection(R11,R2,R3) | ctr(C,A,B,R3)

In the head of the rule (3), the SCI operation has been applied to the first constraint ctr(C,A,B,R1) in order to obtain the constraint ctr(B,C,A,R1'). As the SCI operation is idempotent of level three, the HM operation is applied as part of the guard of the rule to obtain the desired result.

By applying the five operations to the first and second constraints of the head of rule (3) independently, we have defined 11 simplification CHRs to compute the intersection part of operation (2).

The part of the operation (2) which refers to the composition ("⊗") corresponds to the BSIP defined in the previous section. It is implemented by propagation CHRs. For instance:

ctr(C,A,B,R1), ctr(D,B,C,R2) ==> composition(R1,R2,R3) | ctr(D,A,B,R3) (4)

Where composition/3 refers to the set of facts of the PROLOG database which defines the 15×15 inference table shown in figure 3 b.

The lack of completeness of the constraint graph is also supplied by the definition of a total of 11 propagation CHRs, as it happened with the intersection part of operation (2).

4 The use of point and extended objects as primitive of reasoning

In this section, a new approach for representing and reasoning, in a uniform way, with point and extended objects as primitive of reasoning, in those models not based on projections, is presented. Constraint Logic Programming instantiated to Finite Domains —CLP(FD)[1]— (which is also included as a library in [3]) has been used as tool to define the model. This tool permits the definition of variables with a finite and integer domain associated, and linear terms which are linear integer combinations of integer domain variables.

The model is based on:

- The definition of a grid with a context dependent size, which is implemented by two finite domain variables (one for each axis X and Y). All the spatial objects are projected onto the grid. Moreover, the infinite number of points belonging to a qualitative spatial region is simplified to a finite and small number of points, i.e., the physical space is discretized and therefore simplified.
- The two-dimensional objects are represented by two finite domain variables, one for each axis of the grid.
- The straight lines which define the RSs are represented by a set of linear terms. In order to determine the set of linear terms, it is necessary to distinguish the case in which both objects which define the RS are points from the case in which both are extended objects.

The Freksa and Zimmermann's orientation RS is originally defined by two point objects. Each of these point objects are represented by two finite domain variables whose domains contain a unique value. For instance, we can say that an object, *c*, is to the right-front of the orientation RS formed by the two point objects, *a* and *b*. Initially, the domains of the two variables associated to the object *c* will contain all the values of the work space. The set of values will be reduced to the set of points belonging to the grid and to the right-front region after applying the following linear terms (figure 4 a):

(1) *(Bx-Ax)*Cy#<(By-Ay)*(Cx-Ax)+(Bx-Ax)*Ay*
(2) *(By-Ay)*Cy#>(Ax-Bx)*(Cx-Bx)+(By-Ay)*By*

The fifteen qualitative regions defined by the Freksa and Zimmermann's fine orientation RS will be defined with three linear terms at most [7].

When the RS is formed by a point object and an extended object or by two extended objects, instead of a straight line which joins both objects, it will be necessary to define a set of straight lines.

First of all, an extended object is approximated, in our approach, by the smaller rectangle which encloses the object.

The original Freksa and Zimmermann's fine orientation RS, defined by two point objects, is generalized when one of the objects which define the RS is a point and the other one is an extended object, in the following way: a straight line is drawn from the point object to each one of the points, belonging to the grid, which define the extended object, and their perpendicular lines. From all these straight lines, the ones which define the boundaries of each region are considered (figure 4 b), similarly to the Allen's intervals temporal logic [1]. These straight lines are implemented as linear terms. The number of qualitative regions does not change with respect

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1 We refer to [17] for a review of the tool.

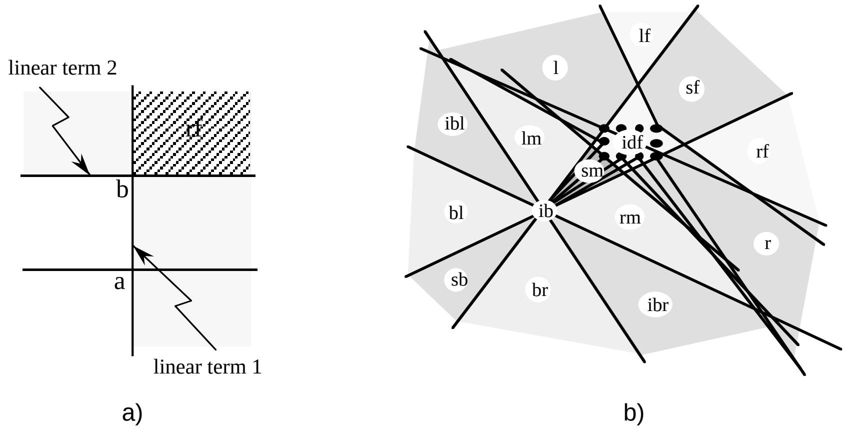


Figure 4 a) The two linear terms which define the right-front qualitative region; b) the orientation RS when *a* is a point object and *b* is an extended object.

to the original RS, although the shape of the regions is different. Some qualitative regions even overlap.

When both objects are extended, the previous method is also valid. The number of qualitative regions remain unchanged, the shape of the regions changes, and the amount of straight lines —and therefore the number of linear terms— increases.

For further details of the representation of point and extended objects as primitive of reasoning in the orientation RS, the positional RS, and the cardinal directions RS, we refer to [7] and [6].

5 Application: a Qualitative Navigation Simulator (QNavSim)

Traditional methods for robot navigation need a precise and reliable metric of the robot trajectory. These methods provide good results in certain, known and indoor environments where all the work space is perceived from any position. However, they fail in other environments [26]. Qualitative navigation is an alternative way to solve the autonomous robot navigation problem, which tries to take profit of uncertain and imprecise spatial information, as human beings do.

The qualitative spatial model described in the previous sections, is used to build, as demonstrator, a qualitative robot navigation simulator, in a structured environment. The aim is the simulation of the movement of a robot through the structured environment of the city of Castellón, from a starting to a goal position, using a set of qualitative spatial relationships as initial information.

The system architecture proposed consists of two parts (figure 5): the spatial reasoning module and the QNavSim module. The spatial reasoning module is formed by a set of constraints solvers which manages the different spatial aspects which have been integrated into the same spatial model; and a set of processors of point and extended objects, one for each spatial aspect considered. This module has been explained in the previous sections for the concept of orientation. In this section, the features of the QNavSim will be explained.

The grid with context dependent size, introduced in the previous section, is used to define the world model which consists of streets and spatial objects or landmarks. The streets are approximated to intersections of the grid and defined as facts of the PROLOG data base. For instance, the fact *street(22,16,23,17)* means that there is a portion of the street from the

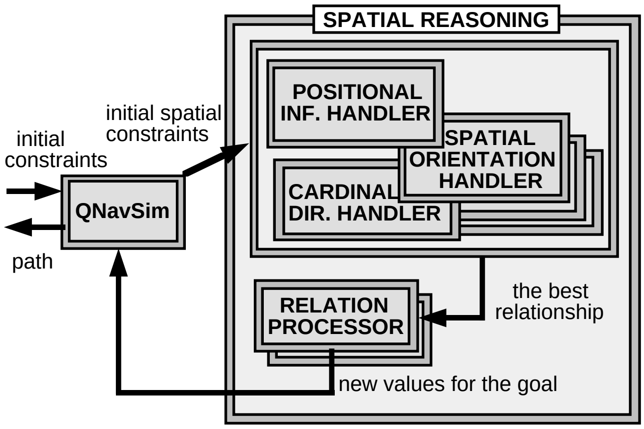


Figure 5. The system architecture for qualitative robot navigation.

intersection, X=22, Y=16, of the grid, to the neighboring intersection, X=23, Y=17. The name and position of any important landmark are also stored as facts of the PROLOG data base. For instance, the fact *position(restaurant,20,17)* means that the restaurant is positioned at X=20, Y=17, corresponding to the grid.

The initial information given to the simulated robot is its initial position in terms of the grid (in the example, it will be at "Plaza La Paz"), the name of the objective (the bull ring) and a set of qualitative spatial relationships. For instance, for orientation information the relationships shown in table 1 will be provided:

Table 1. Set of initial orientation relationships.

(I)	ctr_orient(Bull,Paz,Clara,[lf,l,lm,ibl,bl]),
(II)	ctr_orient(Mayor,Paz,Clara,[lf]),
(III)	ctr_orient(Rey_Don_Jaime,Clara,Mayor,[lm,l,lf]),
(IV)	ctr_orient(Bull,Mayor,Rey_Don_Jaime,[sf]),
(V)	ctr_orient(Station,Clara,Mayor,[lf,l,lm]),
(VI)	ctr_orient(Bull,Mayor,Station,[lm]),
(VII)	ctr_orient(Ribalta,Mayor,Station,[rm]),
(VIII)	ctr_orient(Hospital,Mayor,Station,[lm]),
(IX)	ctr_orient(Bull,Ribalta,Hospital,[sm]),
(X)	ctr_orient(Bull,Rey_Don_Jaime,Ribalta,[l]).

Relationship (I) means that the bull ring is somewhere to the left of the RS formed by Plaza La Paz and Plaza Santa Clara, and so on. It is possible to draw very different cognitive maps which satisfy all the previous constraints. Two of these possibilities are shown in figure 6.

In both cognitive maps, the objective is in a different position —even qualitative very different— with respect to the initial position. This means that, with the initial spatial relationships, the problem of going from a starting position to an objective is not directly solved, i.e. the objective region is not small enough for the robot to directly arrive.

Another feature of the simulator is the knowledge acquisition of the robot. When the robot arrives to any intersection of the grid, the PROLOG data base is looked for to check if a landmark exists at this position.

In summary, the QNavSim algorithm consists of, first of all, a call to the spatial reasoning module, providing as input the initial set of qualitative spatial relationships. The output is the set of relationships inferred from the initial knowledge. The best relationship between the current

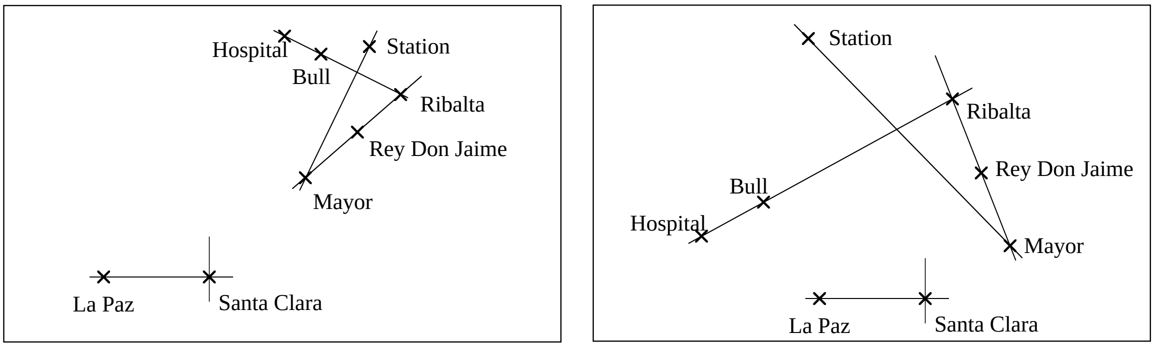


Figure 6. Two cognitive maps which satisfy all the initial orientation constraints.

position and the goal (or in its absence the best relationship between the current position and any subgoal) is used to compute the region where it is possible to find the goal (or subgoal). Then the loop of the algorithm starts. The agent moves one step forward (which corresponds to a grid unit) if there is no crossroad, or it chooses as its next step the one which is "closer" to the goal (or in its absence a subgoal) when there is a crossroad. At every new position, the agent checks whether it exists a landmark there. If there is no landmark, the program control goes back to the beginning of the loop. If a landmark is perceived at this position, it is checked whether the goal (or subgoal) region can be narrowed down by using some inferred spatial relationship, before going back to the beginning of the loop. The algorithm finishes when the robot arrives to the goal. The complete description of the QNavSim algorithm can be found in [6].

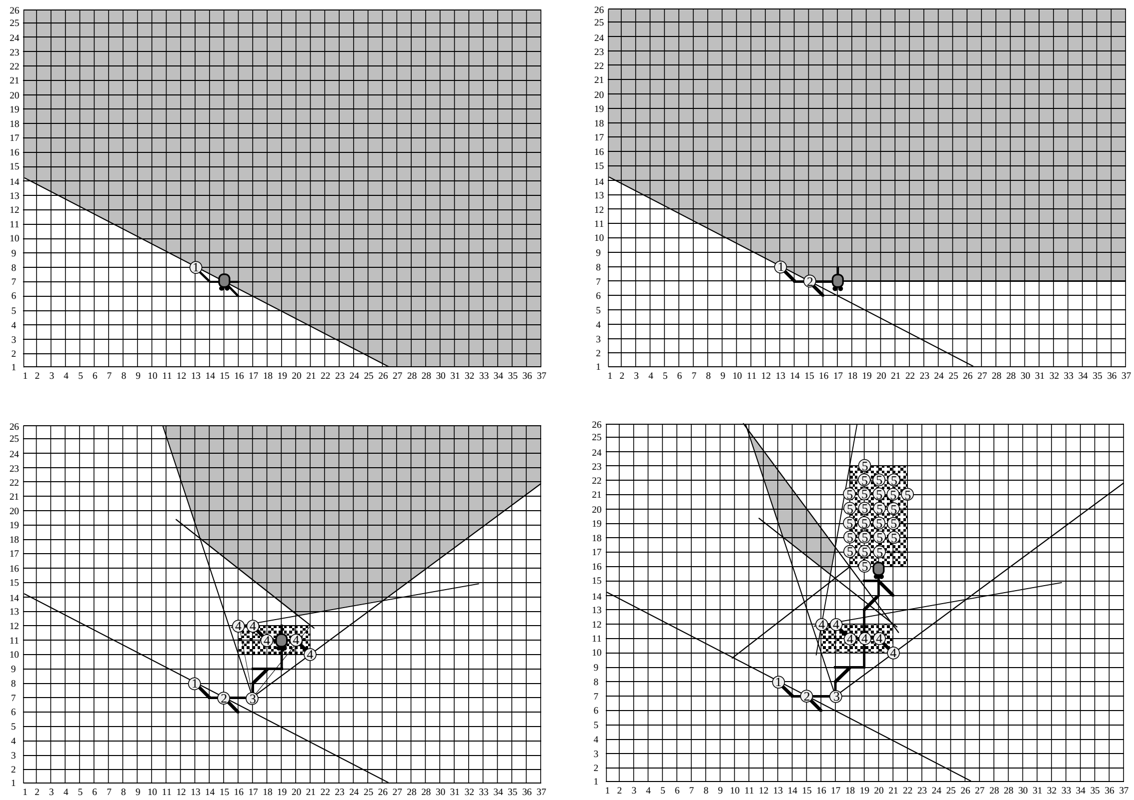
An example of the execution of the QNavSim in different steps is shown in figure 7. As the movement of the robot through the streets is simulated, the acquisition of new knowledge will allow the reduction of the objective region, until it will be small enough for the robot to get the goal.

This is a very nice example of nonmonotonic reasoning, in the sense that when new knowledge is acquired (new landmarks are perceived by the robot), it might invalidate previous conclusions (some of the previous spatial relationships become more restricted). When the robot has perceived all the landmarks implied in the reasoning process, only one cognitive map will remain true, the one which corresponds to reality, the other possible maps will not be valid anymore. The management of nonmonotonic reasoning in our application has been automatically accomplished by means of representing knowledge using constraints and applying consistency techniques [16].

6 Conclusions and future work

The contributions of the work presented in this article are the following:

- The definition of an approach for integrating several aspects of space into the same model. Although in this paper only the orientation aspect has been explained as example of this approach, the concepts of position and cardinal directions have also sucessfully integrated into the spatial model [4,5,6,7,8,9,10].
  For each aspect of space to be integrated we have defined: (1) its representation; (2) the basic step of the inference process (BSIP); and (3) the full inference process (FIP). A uniformity of the implementation of these three parts has allowed the integration. The operations of the algebra provided by the representation as well as the BSIP are implemented as facts of the PROLOG database. Constraint logic programming extended with constraint handling rules



① Paz ② Clara ③ Mayor ④ Rey Don Jaime ⑤ Ribalta

Figure 7. Different steps of the execution of the QNavSim.

(CLP+CHRs) are used to implement a constraint solver (CS) which solves in a straightforward way the FIP for each aspect of space to be integrated. The paradigm CLP+CHRs provides the suitable level of abstraction for the definition of incremental, flexible and general-purpose user-defined CSs for each aspect to be integrated. Another important feature of these CSs is their efficiency —the complexity of each CS is $O\left(n^3\right)$ where n is the number of spatial landmarks considering in the reasoning process—.

- The definition of a new model to deal with point and extended objects as primitive of reasoning in the orientation and position models not based on projections (which are considered more cognitive). Constraint logic programming instantiated to finite domains, CLP(FD), has been used as tool.
- The development of a Qualitative Navigation Simulator (QNavSim) where the movement of a robot through the structured environment of the city of Castellón is simulated.

In this paper, an interpretation of qualitative spatial labels like "right-front" or "close" has been done for a qualitative navigation simulator. The interpretation of the reasoning process for a real mobile robot will be different. This is our current interest. We also propose as future work further integration of another aspects of space into the same framework. Many aspects in the temporal reasoning field have been considered as instances of the constraint satisfaction problem. These qualitative temporal models and the qualitative spatial model presented in this paper, would be a good starting point to study qualitatively the concept of movement. The integration of qualitative naming and comparing distances and imprecise quantitative distances would also be of great interest.

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